Advanced high strength steel springback optimization by projection-based heuristic global search algorithm

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1. Introduction

Nowadays, advanced high strength steels (AHSSs) are widely applied in the automotive industry. AHSSs show advantages in automobile lightweight and crashworthiness due to their low cost, light weight and high strength. A widely concerned problem is that the springback of AHSS is difficult to be controlled [1,2]. Actually, light weight and high strength. A widely concerned problem is that the springback of AHSS is difficult to be controlled [1,2]. Actually, material properties, tool’s geometry and technological parameters are the major factors that significantly influence the springback [3–7]. Reasonable parameter settings can reduce the springback and improve the final product quality. Therefore, a study on springback optimization is essential.

The major challenge of springback optimization is the expensive computational cost of finite element (FE) analysis. In order to improve the efficiency, the most effective method is to use metamodels that can approximate the results of FE analysis. So far, there are several widely used metamodeling techniques, such as polynomial, radial basis function (RBF), support vector regression (SVR) and Kriging. Wang and Shan [8] summarized these common metamodeling techniques used in engineering design and optimization. Recently, various metamodel-assisted springback optimizations have been developed. Naceur et al. [9] adopted the combination of response surface method with gradient searching for springback minimization. Meinders et al. [10] performed sequential approximation optimization (SAO) on Kriging model for minimum springback. Ingarao et al. [11] successfully reduced springback and thinning failure by response surface methodology based multi-objective optimization. Bekar et al. [12] employed two sets of surrogate models to obtain a robust optimum design of tools for springback.

It is well known that the efficiency of metamodel-assisted optimization strongly depends on the accuracy of the metamodel. Unfortunately, with the increasing nonlinearity and dimensionality of the problem, traditional metamodeling techniques can hardly supply an accurate surrogate model. Therefore, a general set of quantitative model assessment and analysis tool termed high-dimension model representation (HDMR) is suggested. The theory of HDMR was introduced for by Rabitz and Omer [13]. It is a divide-and-conquer algorithm that can deduce the behavior of a multivariable input–output system. Similar to Taylor expansion, a HDMR structure can relieve the difficulty for nonlinear approximation of the inputs upon the output. This particular hierarchical structure can relieve the difficulty for nonlinear approximation

\[
f(x) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{1 \leq i < j \leq n} f_{ij}(x_i, x_j) + \cdots + \sum_{1 \leq i_1 < i_2 < \cdots < i_n \leq n} f_{i_1i_2\cdots i_n}(x_{i_1}, x_{i_2}, \ldots, x_n) \quad (1)
\]

Each term represents a unique independent or coupling contribution of the inputs upon the output. This particular hierarchical structure can relieve the difficulty for nonlinear approximation

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problems to some extent. Amongst HDMRs with different features, Cut-HDMR shows obvious advantage in terms of efficiency and computational convenience. It provides a global representation of a high dimensional function using a few regularly distributed samples in the design space. Shan and Wang [14] and Wang et al. [15] respectively combined Cut-HDMR with RBF and moving least square (MLSL) for multivariable problems. In this work, to enhance the efficiency of the optimization, Kriging-based Cut-HDMR is used to approximate the component functions.

Design of experiment (DOE) (sampling strategy) is important to the efficiency of optimization. The classical DOE is off-line sampling pattern, such as factorial design, Latin hypercube sampling (LHS). For off-line pattern, the training samples are pre-determined without a priori knowledge about the objective function. However, for high-dimensional problems, due to lack of direction, such sampling strategy may generate redundant samples and bring extra computational cost. In order to overcome the bottleneck of traditional off-line strategies, various on-line sampling strategies have been developed for reducing the number of evaluations. In contrast of the off-line, on-line strategies make use of the feedback of function evaluations to consistently guarantee the sampling in a right direction. Jones et al. [16] developed DIRECT algorithm for finding the global minimum of a multivariate function. Wang et al. [17] suggested boundary and best neighbor searching algorithm (BBNS) based on Kriging technique and successfully applied it to sheet forming problems. Fuzzy c-mean (FCM) was combined with RSM and applied to crashworthiness optimization by Wang et al. [18]. In this work, a heuristic sampling strategy-mode pursuing sampling (MPS) algorithm is employed to generate new samples. Furthermore, a projection technique is proposed to make MPS strategy match Cut-HDMR.

The rest of this paper is organized as follows. Section 2 introduces basic principles including Kriging-based Cut-HDMR technique, MPS strategy. P-HGS method and its critical details are described in Section 3. A mathematical function is employed to illustrate the optimizing process of P-HGS. In order to verify the feasibility of the method, P-HGS is tested by a few nonlinear mathematical functions in this section. In Section 4, two springback optimization problems are successfully solved by P-HGS. Corresponding discussions are in Section 5. Finally, Section 6 gives the conclusion.

2. Basic theory

2.1. Kriging-based Cut-HDMR technique

Although the Cut-HDMR can represent a nonlinear underlying function in terms of several component functions, a basis is also needed to complete the model. Among popular metamodelling techniques, Kriging interpolation attracts the most attention. Kriging is based on the best unbiased predictor and can adaptively adjust interpolation parameters. Compared with other metamodelling parameters, the methods can be determined more rationally [19]. Thus, a Kriging-based Cut-HDMR technique is suggested.

In a Cut-HDMR expansion, each component function is determined by the output relative to the cut center $x_0$ along corresponding axes, planes, and hyper-planes. The sampling pattern of Cut-HDMR is illustrated in Fig. 1. In the convergence limit, Cut-HDMR is invariant to the location of the cut center.

This paper uses Kriging to approximate the component functions in Eq. (1). The details of Kriging technique are described by Sasena [19]. In this work, the DACE toolbox of MATLAB is employed to construct the Kriging models and a Gaussian correlation function is adopted. Without loss of generality, consider the first-order term $f_i(x_i), (i = 1, 2, \ldots, n)$. Suppose that there are $m_i$ samples $(x_i^1, x_i^2, \ldots, x_i^{m_i})$, and we have:

$$f_i(x_i^k) = f_i\left([x_1^0, x_2^0, \ldots, x_{i-1}^0, x_i^k, x_{i+1}^0, \ldots, x_n^0]\right) - f_0 \quad (k = 1, 2, \ldots, m_i)$$

(2)

where $f_0 = f(x_0)$. Based on $f_i(x_i^1), f_i(x_i^2), \ldots, f_i(x_i^{m_i})$, the Kriging model $f_i(x_i)$ can be constructed. Similarly, for the second-order term $f_{ij}(x_i, x_j)$:

$$f_{ij}(x_i, x_j) = f\left([x_1^0, x_2^0, \ldots, x_{i-1}^0, x_i, x_{i+1}^0, \ldots, x_j, x_{j+1}^0, \ldots, x_n^0]\right) - f_i(x_i) - f_j(x_j) - f_0$$

(3)

Randomly select two points $x_i^k (k = 1, 2, \ldots, m_i)$ and $x_j^l (l = 1, 2, \ldots, m_j)$. If $f_i(x_i^k, x_j^l) = 0$, $f_{ij}(x_i, x_j)$ is supposed not to exist, else $f_{ij}(x_i, x_j)$ should be modeled.

In this paper, the Kriging-based Cut-HDMR (Kriging-HDMR) expands to the second order as following form:

$$\hat{f}(x) \equiv f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{1 \leq i < j \leq n} f_{ij}(x_i, x_j)$$

(4)

where $\hat{f}(x)$ is the Kriging model. Based on the assumption that only low-order component functions have a significant impact upon the output [14], higher order component functions are neglected in Eq. (4). The Kriging-based Cut-HDMR includes all univariate and bivariate component functions and the nonlinear feature of the underlying function can be well captured by them.

2.2. MPS strategy

In order to further improve the efficiency of Kriging-based HDMR, a heuristic sampling strategy is integrated with the Kriging-based HDMR. Theoretically, any space reduction strategies can be used for sampling procedure, such as DIRECT, BBNS, fuzzy c-mean (FCM) and others. In this work, a novel heuristic sampling method, MPS [20] is used. Compared with above mentioned methods, MPS can direct the sampling towards more promising areas without loss of global search. For the details of the MPS, please refer to the reference [20].
3. P-HGS algorithm

3.1. Projection strategy

According to Eq. (4), the Kriging-based HDMR includes a group of Kriging models with different variables. For each Kriging model, there is a special sampling axis or plane. For example, suppose that $x_0 = (x_0^1, x_0^2, \ldots, x_0^n)$ is the cut center; the sampling axis of $\overline{f}_i(x_i)$ is $x_k = x_k^0 (k = 1, 2, \ldots, n, k \neq i)$, and the sampling plane of $f_{ij}(x_i, x_j)$ is $x_l = x_l^0 (l = 1, 2, \ldots, n, l \neq i, j)$. The point on a sampling axis or plane can be used for the corresponding Kriging model, but an arbitrary point probably cannot be utilized. Therefore, the projection strategy is suggested.

Projection strategy is to project an arbitrary point $y = (y_1, y_2, \ldots, y_n)$ in the design space to the sampling axes or planes (Fig. 2). The projection operators $P$ are defined as:

$$P_i(y) = (x_i^0, x_1^0, x_2^0, \ldots, x_{i-1}^0, y_i, x_{i+1}^0, \ldots, x_n^0) \quad (i = 1, 2, \ldots, n)$$

$$P_{ij}(y) = (x_i^0, x_1^0, x_2^0, \ldots, x_{i-1}^0, y_i, x_{i+1}^0, \ldots, x_j^0, y_j, x_{j+1}^0, \ldots, x_n^0) \quad (i < j)$$

According to Eqs. (2) and (3),

$$f_i(y_i) = f(P_i(y_i)) - f_0 \quad (i = 1, 2, \ldots, n)$$

$$f_{ij}(y_i, y_j) = f(P_{ij}(y)) - f(P_i(y)) + f(P_j(y)) + f_0 \quad (i < j)$$

Thus, $f_i(y_i)$ and $f_{ij}(y_i, y_j)$ can be used to update $\overline{f}_i(x_i)$ and $\overline{f}_{ij}(x_i, x_j)$ respectively. Perform all the projection operators on $y$, and the entire Kriging-HDMR can be updated.

3.2. P-HGS strategy

The advantage of Kriging-HDMR is to explicitly represent a multivariable function in terms of a few component functions within a limited number of samples; the advantage of MPS is to heuristically guide the sampling towards more promising areas without loss of global search. For this reason, Kriging-HDMR and MPS are integrated. The P-HGS is to use Kriging-based Cut-HDMR technique to construct the metamodel for MPS based on the projection strategy. Through updating the Kriging-HDMR approximation repeatedly, the sampling process can be gradually guided to more promising region that probably contains the global optimum. But there is a small chance that the optimum can be exactly reached. Thus, in the proposed method, local optimization is added to exploit the global optimum.

The flowchart of the P-HGS is presented in Fig. 3. The details of the P-HGS are described in follows (iter is current iteration):

1. **Start**
2. **Build original Kriging-HDMR approximation $f(x)$**
3. **Construct sampling guidance function $g(x)$ and generate a discrete point set $S_c(g)$**
4. **Deduce the cumulative distribution ($\hat{G}_1, \hat{G}_2, \ldots, \hat{G}_k$)**
5. **Draw a new sample according to the cumulative distribution**
6. **Perform each projection operator on the new sample**
7. **Obtain the projected point on each axis or plane**
8. **Delete the overlapped points**
9. **Reconstruct each Kriging model**
10. **Search the local optimum of current Kriging-HDMR model**
11. **Local optimum**
12. **Convergence?**
   - **Yes**
   - **end**
   - **No**

**Fig. 2.** An illustration of projection strategy.

**Fig. 3.** Flowchart of the P-HGS.
Step 1. Choose the center of the design space as the cut center, evaluate it and we have \( f_0 \). For each dimensionality \( i \), sample two bound points \( x_{iL}, x_{iR} \) and evaluate them.

Step 2. Construct original Cut-HDMR approximation \( \hat{f}(\mathbf{x}) \). Initialize \( \text{iter} = 0 \).

Step 3. Set \( \text{iter} = \text{iter} + 1 \). Define the sampling guidance function \( g(\mathbf{x}) = \max(\hat{f}(\mathbf{x})) - f(\mathbf{x}) \); generate a discrete set \( S_0(g) \) consisting of \( N \) uniformly distributed points (\( N \) is usually large).

Step 4. Sort the points of \( S_0(g) \) in descending of their \( g(\mathbf{x}) \) values and assign them into \( K \) contours \( \{ E_1, E_2, \ldots, E_K \} \) with equal size. A discrete probability distribution \( \{ P_1, P_2, \ldots, P_K \} \) proportional to the average \( g(\mathbf{x}) \) values over the \( K \) contours can be obtained.

Step 5. Build the cumulative distribution \( \{ G_1, G_2, \ldots, G_K \} \) based on \( \{ P_1, P_2, \ldots, P_K \} \). In this work, a modified cumulative distribution \( \{ \bar{G}_1, \bar{G}_2, \ldots, \bar{G}_K \} \) is adopted to accelerate convergence.

Step 6. A new sample is drawn from the set \( S_0(g) \) according to the cumulative distribution \( \{ \bar{G}_1, \bar{G}_2, \ldots, \bar{G}_K \} \) and the uniform distribution within each contour; evaluate the new sample.

Step 7. Perform all projection operators \( P \) on the new sample and obtain the projected points on the sampling axes and planes; delete the overlapped points. Evaluate these projected points, and update all Kriging approximations.

Step 8. Start with the new sample, and perform trust-region-reflective method on current Kriging-HDMR model. Find and evaluate the local optimum.

Step 9. If

\[
\begin{align*}
\text{iter} > 1 \quad & \frac{|\text{iter} - \text{iter}_{\text{min}}|}{\text{iter}_{\text{min}}} \leq \eta, \quad \eta \in (0, 1) \\
\end{align*}
\]

the procedure ends, if not, proceeds to Step 10, where threshold \( \eta \) can be set by user and the default value is 5%.

Step 10. Perform all projection operators \( P \) on the local optimum obtained in Step 8 and obtain the projected points on the sampling axes and planes; delete the overlapped points.

Step 11. Evaluate these projected points, and update all Kriging approximations and goes back to Step 3.

To clarify P-HGS clearly, the optimization procedure of a popular 2d function-six camel back (SC) function is illustrated. The explicit expression of SC function is:

\[
\begin{align*}
V_1 &= -x_1^2 + 2x_1 \quad \text{for } x_1 \in [0, 1] \\
V_2 &= (x_1 - 2)^2 + 2x_2 \quad \text{for } x_2 \in [0, 1] \\
V_3 &= (x_1 - 4)^2 + (x_2 - 1)^2 \quad \text{for } x_1, x_2 \in [0, 1] \\
V_4 &= (x_1 - 6)^2 + (x_2 - 1)^2 \quad \text{for } x_1, x_2 \in [0, 1] \\
V_5 &= (x_1 - 7)^2 + (x_2 - 1)^2 \quad \text{for } x_1, x_2 \in [0, 1] \\
V_6 &= (x_1 - 8)^2 + (x_2 - 1)^2 \quad \text{for } x_1, x_2 \in [0, 1] \\
\end{align*}
\]
Test mathematical functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Name</th>
<th>Expression</th>
<th>Interval</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F10 (10d)</td>
<td>( f(x) = \sum_{i=1}^{10} (x_i^2 + 1) )</td>
<td>( x \in [0, 1] )</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>F16 (16d)</td>
<td>( f(x) = \sum_{i=1}^{16} (x_i^2 + 1) (x_i^2 + x_i + 1) )</td>
<td>( x \in [-1.4, -0.4] )</td>
<td>25.887</td>
</tr>
<tr>
<td>3</td>
<td>Rastrigin (5d)</td>
<td>( f(x) = \sum_{i=1}^{5} (x_i^2 - 10 \cos(2\pi x_i)) + 10 )</td>
<td>( x \in [-0.2, 0.8] )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Goldstein price (2d)</td>
<td>( f(x) = 1 + 10(x_1^2 + x_2^2) + 10(1 - \cos(2\pi x_1))(1 - \cos(2\pi x_2)) )</td>
<td>( x \in [-2, 2] )</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>SC (2d)</td>
<td>( f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4 )</td>
<td>( x \in [-2, 2] )</td>
<td>-1.0316</td>
</tr>
</tbody>
</table>
3.3.2. Benchmarks

In order to verify the performance of the proposed P-HGS, several nonlinear functions (Table 2) are tested using the P-HGS and MPS [20] respectively. The accuracy of the optimization is indicated by minimum function value (Min); number of function evaluations (NFEs) indicates the efficiency of the optimization. To obtain representative results, each function is carried out for 10 times. The averages and standard deviations are presented in Table 3. To illustrate the advantage of Kriging-HDMR in accuracy, each Kriging-HDMR model is accompanied by a Kriging model based on the same samples. For each function, we use extra 10,000 random points to calculate the c. The average values of the 10 runs are listed in Table 4.

Table 3
Comparison of the P-HGS and MPS method.

<table>
<thead>
<tr>
<th>Function</th>
<th>Method</th>
<th>Min (average)</th>
<th>NFEs Average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P-HGS</td>
<td>0.0059</td>
<td>137.6000</td>
<td>14.7119</td>
</tr>
<tr>
<td></td>
<td>MPS</td>
<td>0.1198</td>
<td>436.4000</td>
<td>167.3829</td>
</tr>
<tr>
<td>2</td>
<td>P-HGS</td>
<td>25.9044</td>
<td>268.4000</td>
<td>12.7138</td>
</tr>
<tr>
<td></td>
<td>MPS</td>
<td>28.4085</td>
<td>343.7000</td>
<td>85.9119</td>
</tr>
<tr>
<td>3</td>
<td>P-HGS</td>
<td>0.7551</td>
<td>61.2000</td>
<td>5.2877</td>
</tr>
<tr>
<td></td>
<td>MPS</td>
<td>0.8204</td>
<td>126.6000</td>
<td>35.8419</td>
</tr>
<tr>
<td>4</td>
<td>P-HGS</td>
<td>52.8459</td>
<td>24.8000</td>
<td>7.3221</td>
</tr>
<tr>
<td></td>
<td>MPS</td>
<td>104.3931</td>
<td>62.9000</td>
<td>21.4590</td>
</tr>
<tr>
<td>5</td>
<td>P-HGS</td>
<td>-1.0277</td>
<td>28.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>MPS</td>
<td>-1.0191</td>
<td>37.2000</td>
<td>7.8588</td>
</tr>
</tbody>
</table>

Table 4
Comparison of the accuracy of Kriging-HDMR and Kriging.

<table>
<thead>
<tr>
<th>Function</th>
<th>Method</th>
<th>$R^2$</th>
<th>RAAE</th>
<th>RMAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kriging-HDMR</td>
<td>0.9938</td>
<td>0.0576</td>
<td>0.5984</td>
</tr>
<tr>
<td></td>
<td>Kriging</td>
<td>-0.0693</td>
<td>0.7827</td>
<td>4.7810</td>
</tr>
<tr>
<td>2</td>
<td>Kriging-HDMR</td>
<td>0.9963</td>
<td>0.0409</td>
<td>0.4428</td>
</tr>
<tr>
<td></td>
<td>Kriging</td>
<td>-0.9007</td>
<td>1.1001</td>
<td>5.3732</td>
</tr>
<tr>
<td>3</td>
<td>Kriging-HDMR</td>
<td>1.0000</td>
<td>0.0032</td>
<td>0.0139</td>
</tr>
<tr>
<td></td>
<td>Kriging</td>
<td>0.6081</td>
<td>0.4751</td>
<td>2.3489</td>
</tr>
<tr>
<td>4</td>
<td>Kriging-HDMR</td>
<td>0.6412</td>
<td>0.3164</td>
<td>4.4733</td>
</tr>
<tr>
<td></td>
<td>Kriging</td>
<td>-1.4000</td>
<td>0.6280</td>
<td>9.0060</td>
</tr>
<tr>
<td>5</td>
<td>Kriging-HDMR</td>
<td>0.9933</td>
<td>0.0525</td>
<td>0.2170</td>
</tr>
<tr>
<td></td>
<td>Kriging</td>
<td>0.9558</td>
<td>0.1265</td>
<td>0.8962</td>
</tr>
</tbody>
</table>

3.3.2. Benchmarks

In order to verify the performance of the proposed P-HGS, several nonlinear functions (Table 2) are tested using the P-HGS and MPS [20] respectively. The accuracy of the optimization is indicated by minimum function value (Min); number of function evaluations (NFEs) indicates the efficiency of the optimization. To obtain representative results, each function is carried out for 10 times. The averages and standard deviations are presented in Table 3. To illustrate the advantage of Kriging-HDMR in accuracy, each Kriging-HDMR model is accompanied by a Kriging model based on the same samples. For each function, we use extra 10,000 random points to calculate the c. The average values of the 10 runs are listed in Table 4.

According to Table 3, for each function, the Min value of P-HGS is better than that of MPS. Actually, except Goldstein price, each Min of P-HGS is very close to the theoretical minimum. For Goldstein price function, the function value varies severely around the global optimum (0, -1); the P-HGS can find the neighborhood of the global optimum, but the theoretical minimum can be hardly reached. Table 3 also shows that the efficiency and robustness of P-HGS are both much better than that of MPS. For example, it averagely takes 436.4 function evaluations for MPS to optimize the F10, while the P-HGS only costs 137.6 function evaluations to obtain better optimum.

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Fig. 8. An illustration of tools for case I.

Fig. 9. An illustration of springback [21].

Fig. 10. Measurement for springback [21].
According to Table 4, the Kriging-HDMR model has higher accuracy than the Kriging model, especially for high-dimensional functions. For F10, F16 and Rastrigin, the values of $R^2$ for Kriging-HDMR are very close to 1, while the maximum (best) value for Kriging is 0.6081; the values of $RAAE$ for Kriging-HDMR are all less than 0.1, while the minimum (best) value for Kriging is 0.4751; the values of $RMAE$ for Kriging-HDMR are all less than 1, whereas the minimum (best) for Kriging is 2.3489.

Summarily, the proposed P-HGS is verified a high performance global optimization algorithm.

4. Springback optimization

4.1. Case I: blank holder force optimization

4.1.1. Problem descriptions

The U-shaped part presented in Numisheet93 is suggested (see Fig. 8). Despite of the simplicity, the case is a benchmark problem.
for isotropic springback analysis and is very representative. The springback of the U-shaped part is illustrated in Fig. 9. In this work, a standard method for springback assessment [21] is adopted (Fig. 10). The \( o-a \) section is constrained and the shape of the product in \( x-z \) direction after springback is curve \( o-a-c-b-d-e-f \) (before springback curve \( o-a-c-b-d-e-f \) is straight and vertical to the \( o-x \) axis). Curve \( a-c-b-d \) can be approximately seen as a section of a circle with radius \( r \). The curvature of the circle \( 1/r \) is regarded as the objective function. The minimization of the objective function lets to attenuate the springback.

In sheet forming process, the material flow potentially influences the springback. In order to control the material flow more subtly, BHFs changing with time in several stages (Fig. 11) are adopted. The values of BHFs and corresponding key times are design variables. Their constraints and initial values are presented in Table 5.

A popular commercial software DYNAFORM is used to establish the FE model (Fig. 12). All tools are modeled as rigid shell elements. Quadrilateral shell elements are applied in the blank (in 1 mm size) and the blank is modeled as isotropic AHSS DP600. The basic isotropic von Mises yield criteria is adopted and the yielding function [21] is:

\[
f(\sigma) = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\sigma_{xy}^2} = S
\]

where \( S \) is the yield stress and \( \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \) are the Cauchy stress components. The stress–strain curve is fitted by following form:

\[
\sigma = K(e_0 + \varepsilon_p)^n
\]

where \( K \) and \( n \) are respectively 1st hardening coefficient and 2nd hardening coefficient; \( e_0 \) and \( \varepsilon_p \) are the yield strain and effective plastic strain respectively. Material and other corresponding parameters are listed in Table 6.

4.1.2. Optimization and results

The computational time of FE analysis is about 0.1h. The optimization results are listed in Table 7. The variation of \( 1/r \) in the optimization process is illustrated in Fig. 13; after 49 FE evaluations, there is no improvement of the function value. A comparison of
the part in x-z direction before and after optimization is shown in Fig. 14. It can be clearly observed that the optimum design of BHFs reduces the springback obviously.

4.2. Case II: drawbeads and blank holder force optimization

4.2.1. Problem descriptions

In this section, a twist rail [22] is investigated (Fig. 15). The formed workpiece is shown in Fig. 16. In this work, the twist springback [22] is considered. Actually, the twist springback is brought by the torsion movement in cross-sectional direction of the workpiece (Fig. 17). In order to effectively assess the twist springback, the measuring method used by Li et al. [22] is suggested (Fig. 18). The desire shapes of the two cross-sections ($S_1$ and $S_2$) are represented by the “initial” line. After springback, there is a respective angle deviation ($\theta_1$, $\theta_2$) for each cross-section. The angle deviation in unit distance between $S_1$ and $S_2$ is used to assess the twist springback, shown as the following equation:

$$\theta_{ts} = \frac{\Delta \theta}{L} = \frac{\theta_1 - \theta_2}{L_1 + L_2} \quad (16)$$

where the plus-minus properties of $\theta_1$ and $\theta_2$ are considered. In this work, clockwise direction is minus, and the inverse direction is plus.

The twist springback is significantly influenced by the drawbead depth [22]. In FE analysis, there are two various patterns to realize drawbead design. One is to mold actual drawbead geometry on the tools. This requires a blank fairly well meshed near the drawbead and will increase the computational cost. Another way is to establish an equivalent drawbead model in which the actual drawbead is replaced by restraining force assigned to corresponding nodes in regular mesh. In this way, the requirement of a greatly fine mesh for blank is avoided and much computational cost can be saved. In this work, the four equivalent drawbead restraining forces (EDRFs) (Fig. 19) and the BHF are the design variables. The constraints and initial values are listed in Table 8.

Li et al. [22] validated the reliability of FE analysis by comparing the two cross-sections ($S_1$ and $S_2$) after springback obtained from try-out experiment and LSDYNA simulation respectively. This is very beneficial to the following work. In this work, the blank is modeled as quadrilateral shell elements (in 3 mm size), and the tools are modeled as rigid shell elements (Fig. 20). For springback prediction, the three $P_1$, $P_2$ and $P_3$ (see Fig. 15) points are employed to suppress the rigid body motion of the workpiece; $P_1$, $P_2$ have the same x and z coordinates; $P_1$, $P_3$ have the same y and z coordinates. The rigid translations in x, y and z directions of $P_1$ are constrained.

### Table 9

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbolic</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus</td>
<td>$E$</td>
<td>207</td>
<td>GPa</td>
</tr>
<tr>
<td>Material density</td>
<td>$\rho$</td>
<td>7830</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Poisson coefficient</td>
<td>$\nu$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Strength coefficient</td>
<td>$k$</td>
<td>1333.6638</td>
<td></td>
</tr>
<tr>
<td>Strength exponent</td>
<td>$n$</td>
<td>0.2366</td>
<td></td>
</tr>
<tr>
<td>Anisotropy coefficients</td>
<td>$R_0$</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_45$</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_90$</td>
<td>1.58</td>
<td></td>
</tr>
<tr>
<td>Initial thickness of the blank</td>
<td>$h_0$</td>
<td>1.5</td>
<td>mm</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>$\mu$</td>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

### Table 10

Results of optimization of Case II.

<table>
<thead>
<tr>
<th>Optimization results</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHF [kN]</td>
<td>700</td>
</tr>
<tr>
<td>EDRF$_1$ (N/mm)</td>
<td>900</td>
</tr>
<tr>
<td>EDRF$_2$ (N/mm)</td>
<td>635.3</td>
</tr>
<tr>
<td>EDRF$_3$ (N/mm)</td>
<td>800</td>
</tr>
<tr>
<td>EDRF$_4$ (N/mm)</td>
<td>928.3</td>
</tr>
<tr>
<td>Value of objective function: $\theta_{ts}$ (d/mm)</td>
<td>0.0008</td>
</tr>
<tr>
<td>$\theta_1$ (d)</td>
<td>0.2329</td>
</tr>
<tr>
<td>$\theta_2$ (d)</td>
<td>-0.0074</td>
</tr>
<tr>
<td>Number of FE evaluation</td>
<td>67</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9621</td>
</tr>
<tr>
<td>RAAE</td>
<td>0.0325</td>
</tr>
<tr>
<td>RMAE</td>
<td>1.3487</td>
</tr>
</tbody>
</table>

Fig. 21. Variation of $\theta_{ts}$. 

Fig. 20. Finite element model of case II.
while the rigid rotations along x and z axes of \( P_2 \) and the rigid rotation along y direction of \( P_3 \) are constrained.

Anisotropic AHSS DP800 is applied in blank. The following anisotropic yield criterion [23] is adopted:

\[
2\sigma_y^6 = a[K_1 + K_2]^6 + a[K_1 - K_2] + c[2K_2]^6
\]

where \( K_i (i = 1, 2) \) are calculated as follows:

\[
K_1 = \frac{1}{2}(\sigma_x + h\sigma_y)
\]

\[
K_2 = \sqrt{\frac{1}{2}(\sigma_x - h\sigma_y)^2 + p^2\tau_y^2}
\]

In Eqs. (17) and (18), \( a, c, h, p \) are constants related to the anisotropic coefficients \( R_{00}, R_{45}, R_{90} \). The relationship between the stress and strain is same to that in Eq. (15). Material and other corresponding parameters are listed in Table 9.

4.2.2. Optimization and results

It takes about 0.3667\( h \) to perform the FE analysis. The optimization results are shown in Table 10. The variation of \( \theta_1 \) in the optimization process is illustrated in Fig. 21; after 36 FE evaluations, there is no improvement of the function value. The variations of

![Fig. 22. Variation of \( \theta_1 \) and \( \theta_2 \).](image)

![Fig. 23. Forming result of the optimum.](image)

![Fig. 24. Comparison of the twist springback before and after optimization.](image)

![Fig. 25. Cross-sections S1 and S2.](image)
θ₁ and θ₂ are both presented in Fig. 22. According to Fig. 22, θ₁ and θ₂ have the closest values to each other at the optimum, indicating the smallest twist springback. The FLD of final formed blank is presented in Fig. 23. Comparisons of the profiles of the smallest twist springback. The FLD of final formed blank is presented in Fig. 23. After optimization, the torsional movements in cross-section S₁ and S₂ before and after optimization are illustrated in Fig. 24. It can be clearly observed that after optimization, the torsional movements in cross-section S₁ and S₂ both decrease obviously.

5. Discussion

Compared with the commonly used optimization method, the distinctive characteristic of this work is to analysis the correlation amongst the design variables. Obviously, if design variables of springback can be uncoupled, the complex problem can be divided into several cases and the optimization procedure thereby can be implemented in the easy way. Another important issue in this work is to use projection strategy connect the online sampling method, such as MPS with Kriging-based HDMR. With the assistance of the online sampling strategy, Kriging-HDMR is easy to search suitable samples to construct high accuracy model.

As mentioned in Section 4, the proposed heuristic global optimization algorithm called P-HGS is applied to springback problems. Similar to the mathematical benchmarks, R², RMAE and RMAE are used to estimate the validation of the metamodels, shown in Tables 7 and 10 respectively. Due to the expensive computational cost, five test samples are calculated for each metamodel.

For the U-shaped part, Chirita [24] investigated the effect of variable blank holder force on the springback. According to his research, the use of variable BHF that starts with a small value followed by a higher value may reduce the springback [24]. The conclusion shows an agreement with the optimization result in Table 7. However, in his work, the coupling relationships of the BHF and the key times were not discussed. In this work, based on the Cut-HDMR theory, the coupling intensities among the design variables are given out. Coupling intensity indicates the coupling contribution of two variables to the output response. In order to objectively reflect the coupling intensity between two design variables, a coupling coefficient is defined as:

\[
c_{ij}(x) = \frac{1}{m_j} \sum_{k=1}^{m_j} \frac{f_i(x_k) - f_j(x_k)}{f_0(x_k)}
\]

where \(x = x_1, x_2, \ldots, x_{m_j}\) are the samples used to approximate \(f(x)\). Larger value of \(c_{ij}\) indicates higher coupling intensity of \(x_i\) and \(x_j\). The c values of Case I are shown in Fig. 25. According to Fig. 25, F₂ and F₃ has the largest coupling intensity \((c = 0.5028)\) among all pairs of variables; F₁ and t₁ \((c = 0.0612)\), F₁ and t₂ \((c = 0.0612)\), F₁ and t₃ \((c = 0.0612)\), F₁ and t₄ \((c = 0.0758)\) have small coupling intensities. Therefore, it is easy to observe that the correlations of t₁ with other design variables are weak. Actually, it can be analyzed independently.

For the twist rail, Li et al. [22] studied the influence of drawbead depth to the twist springback. Four different drawbead depths were considered in his work. It suggested that the twist springback decreased with the increasing drawbead depth [22]. This conclusion was incomplete, for the reason that uniform drawbead depths were adopted in all drawbeads and the coupling correlations of different drawbeads were not considered. In this work, independent equivalent drawbead restraining forces are applied in various drawbeads. The coupling intensities of all pairs of variables are calculated, shown in Fig. 26. It can be clearly seen that BHF and EDRF₁ \((c = 0.4348)\), BHF and EDRF₂ \((c = 0.3306)\), BHF and EDRF₄ \((c = 0.3209)\) have high coupling intensities; the c values of EDRF₂, EDRF₂ and EDRF₄, EDRF₂ and EDRF₃, EDRF₃ and EDRF₄ are all less than 0.2, indicating relatively low coupling intensities. The coupling analysis discloses the essence of the problem. For example, the coupling coefficient C of EDRF₁ and EDRF₁ is only 0.1263. This implies that drawbead1 and drawbead2 have a weak coupling contribution to the twist springback. Thus, they can be approximately considered to be independent to each other in future research.

6. Conclusion

In the present paper, a heuristic global optimization algorithm called P-HGS is proposed for springback problems. Two typical AHSS springback problems are investigated by the proposed algorithm. It is proved that the P-HGS shows a potential for the springback and also other nonlinear problems. From the numerical results, the following conclusions are generalized:
In the P-HGS, the projection strategy helps to combine the Kriging-based Cut-HDMR with MPS method seamlessly. Tested by several nonlinear functions, the P-HGS improves the accuracy of the metamodel and significantly reduces the number of evaluations in the optimizing process.

The springback of U-shaped part is used to verify the feasibility of the proposed method. The design variables are selected as the BHF in three phases and the two key times. According to the coupling analysis, the BHF and BHF show the most coupling intensity among the variables. Based on the Kriging-HDMR model, considerable reduction is obtained for the value of springback curvature compared with the result before optimization.

The drawbead design is studied for the twist springback of the twist rail. The four EDRFs and the BHF are design variables. It is observed that the effects of drawbead1 and drawbead2 upon the twist springback are approximately independent. Thus, they can be separately considered in the future research. The optimum design decreases both the springback angles \( \theta_1 \) and \( \theta_2 \) of cross-section \( S_1 \) and \( S_2 \).

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